

# Hydromagnetic instability in plane Couette flow

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We study the stability of a compressible magnetic plane Couette flow and show that compressibility profoundly alters the stability properties if the magnetic field has a component perpendicular to the direction of flow. The necessary condition of an instability can be satisfied in a wide variety of flows in laboratory and astrophysical conditions. The instability can operate even in a very strong magnetic field which entirely suppresses other magnetohydrodynamics (MHD) instabilities. The growth time of this instability can be rather short and reach  $\sim 10$  shear time scales.

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## I. INTRODUCTION

Instabilities of the magnetized shear flows play an important role in enhancing transport processes in various astrophysical bodies and laboratory experiments. Shear flows are rather common in astrophysics, and the study of their stability properties is important for the understanding of many phenomena in stars, jets, accretion flows, galaxies, etc.

Likely, the best studied shear flow is differential rotation. It is well-known since the classical papers by Velikhov [1] and Chandrasekhar [2] that a differentially rotating flow with a negative angular velocity gradient and a weak magnetic field can be subject to the magnetorotational instability. This instability has been analyzed in detail in several astrophysical contexts [3–5] because it can be responsible for transport of the angular momentum in various objects. In accretion disks, this instability is also well-studied by numerical simulations in both linear and nonlinear regimes (see, e.g., [6–8]). Astrophysical applications raise great interest in trying to study this instability in the laboratory [9–11]. The experiments, however, are complicated because very large rotation rates should be achieved.

The plane Couette flow is another example of shear flows well-studied in laboratory conditions. The pioneering work on the stability of a magnetized plane Couette flow has been done by Velikhov [12] who obtained that a longitudinal magnetic field with the strength  $\geq 0.1 V_0 \sqrt{4\pi\rho}$  has to stabilize the flow;  $V_0$  is the velocity in the center of the channel and  $\rho$  is the fluid density. A sufficient condition of the ideal instability in a parallel magnetic field has been considered by Chen and Morrison [13]. They argued that the magnetic field can provide a destabilizing effect such as the flow, which is stable in the absence of a magnetic field, can be driven unstable by a relatively weak magnetic field. Also, they found that although strong magnetic shear can stabilize shear flow, there exists a range of magnetic shear that causes destabilization. The linear stability properties of dissipative shear flow in a parallel magnetic field have been considered by Lerner and Knobloch [14]. The authors argued that misaligned linear perturbations can exhibit enhanced decay in such dissipative flows. Stability of incompressible flow in a transverse magnetic field has been studied by Takashima [15,16] who found

that there exist both the stationary and traveling modes of instability.

Note that many previous stability analyses have adopted the Boussinesq approximation and have therefore neglected the effect of compressibility. This is allowed if the magnetic field strength is essentially subthermal, and the sound speed is much greater than the Alfvén velocity,  $c_s \gg c_A$  but often this cannot be realized in real astrophysical conditions and in many numerical simulations. As it was shown by Bonanno and Urpin [17], the compressibility profoundly alters the stability properties of shear flows. The number of new instabilities may occur in a compressible flow if the magnetic field has a component perpendicular to the flow. Bonanno and Urpin [17] have considered the particular case of differentially rotating flows but, likely, the shear-driven instabilities are typical for other shear flows as well. In this paper, we show that the same sort of magnetohydrodynamics (MHD) instabilities can occur also in a plane Couette flow if the magnetic field has a transverse component. The instability can arise even in a sufficiently strong magnetic field that suppresses other MHD instabilities. Stability analysis done in this paper will hopefully prove to be a useful guide in understanding various numerical simulations that explore the nonlinear development of instabilities and their effects on the resulting turbulent state of shear flows.

## II. BASIC EQUATIONS

Consider a plane Couette flow with the velocity  $\vec{V} = V(z)\vec{e}_y$ , where  $x$ ,  $y$ , and  $z$  are the Cartesian coordinates and  $\vec{e}_x$ ,  $\vec{e}_y$ , and  $\vec{e}_z$  are the unit vectors. For the sake of simplicity, we assume shear to be linear,  $V(z) = V_0 + zV'$ , where  $V_0$  and  $V'$  are constant.

We restrict ourselves to an inviscid fluid. The equations of compressible MHD read in this case

$$\vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{\vec{F}}{\rho}, \quad (1)$$

$$\dot{\rho} + \vec{\nabla} \cdot (\rho\vec{v}) = 0, \quad (2)$$

$$\dot{p} + \vec{v} \cdot \vec{\nabla}p + \gamma p \vec{\nabla} \cdot \vec{v} = 0, \quad (3)$$

$$\vec{B} - \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = 0, \quad (4)$$

$$\vec{\nabla} \cdot \vec{B} = 0. \quad (5)$$

Our notation is as follows:  $\rho$  and  $\vec{v}$  are the density and fluid velocity, respectively;  $p$  is the gas pressure;  $\vec{B}$  is the magnetic field,  $\eta$  is the magnetic diffusivity, and  $\gamma$  is the adiabatic index;  $\vec{F}$  is a scalar force introduced in order to provide hydrostatic equilibrium in the basic state. For the sake of simplicity, the flow is assumed to be isothermal.

The basic state on which the stability analysis is performed is assumed to be quasistationary with the magnetic field that has nonvanishing components in all directions,  $\vec{B} = (0, B_y(z), B_z)$ . Generally, a quasistationary basic state in such a magnetic shear flow can be achieved only if dissipative effects are taken into account. In the basic state, the magnetic field  $\vec{B}$  should satisfy the stationary induction equation

$$-\eta \Delta \vec{B} = \vec{e}_y V' B_z. \quad (6)$$

Since  $\vec{B}$  depends only on the  $z$  coordinate, we have

$$\frac{d^2 B_y}{dz^2} = -\frac{V' B_z}{\eta}. \quad (7)$$

Integrating this equation, we obtain

$$B_y = -\frac{V' B_z z^2}{2\eta} + B_{0y} z + B_{0y}, \quad (8)$$

where  $B_{0y}$  and  $B_{0y}$  are constant. We can choose the boundary conditions in such a way that  $B_{0y} = 0$  that corresponds to the absence of electric currents at the low boundary  $z=0$ . We will assume that the longitudinal magnetic field at the low boundary is much stronger than  $V' B_z d^2 / 2\eta$ , where  $d$  is the thickness of the Couette flow. Then,  $B_y \approx B_{0y}$ , and one can neglect the change of  $B_y$  across the basic flow when considering the behavior of small perturbations.

We assume also that the basic state satisfies the condition of hydrostatic equilibrium in the  $x$  and  $z$  directions. In the  $x$  direction, this condition is satisfied always. For the chosen magnetic field, hydrostatic equilibrium in the  $z$  direction yields

$$\frac{dp}{dz} + \frac{1}{4\pi} \frac{d}{dz} (B_y^2) + F_z = 0. \quad (9)$$

Equation (9) can be satisfied if  $p = \text{const}$  and the vertical component of the Lorentz force is balanced by a scalar force in the basic state,

$$F_z = \frac{1}{4\pi} \frac{dB_y^2}{dz}. \quad (10)$$

Note that, generally, there is no hydrostatic equilibrium in the  $y$  direction in our model, but departures from equilibrium are small and can lead only to a very slow change of the basic state. For example, the Lorentz force changes the basic velocity profile  $V(z)$  in accordance with the  $y$  component of the momentum equation,

$$\dot{v}_y = \frac{B_z B'_y}{4\pi\rho} = -\frac{V' B_z^2}{4\pi\rho} = -\frac{c_{Az}^2 V' z}{\eta}, \quad (11)$$

where  $c_{Az}^2 = B_z^2 / 4\pi\rho$ . Integrating this expression, we obtain

$$v_y(t) \approx V(z) - \frac{c_{Az}^2 V' z t}{\eta}. \quad (12)$$

The Lorentz force changes essentially the initial velocity profile on the time scale

$$\tau_{0y} \sim \frac{\eta V(z)}{c_{Az}^2 V' z} \sim \frac{\eta}{c_{Az}^2} \quad (13)$$

[we assume  $V(z) \sim z V'$ ]. Therefore one can neglect this departure from hydrostatic equilibrium if the growth rate of instability,  $\sigma$ , is greater than  $1/\tau_{0y}$ , or

$$\sigma \gg \frac{c_{Az}^2}{\eta}. \quad (14)$$

Under this condition, the chosen basic state can be considered as quasistationary. We will show that this condition is satisfied in many cases of interest.

We consider the stability of perturbations with the space-time dependence  $\propto f(z) \exp(\sigma t)$ . Small perturbations will be indicated by subscript 1, while unperturbed quantities will have no subscript. Then, the linearized MHD equations read

$$\begin{aligned} \sigma \vec{v}_1 + \vec{e}_y V' v_{1z} &= \frac{\rho_1}{\rho^2} \left[ \vec{\nabla} p - \frac{1}{4\pi} (\vec{\nabla} \times \vec{B}) \times \vec{B} + \vec{F} \right] - \frac{\vec{\nabla} p_1}{\rho} \\ &+ \frac{1}{4\pi\rho} [(\vec{\nabla} \times \vec{B}_1) \times \vec{B} + (\vec{\nabla} \times \vec{B}) \times \vec{B}_1], \end{aligned} \quad (15)$$

$$\sigma \rho_1 + \vec{\nabla} \cdot (\rho \vec{v}_1) = 0, \quad (16)$$

$$\sigma p_1 + \gamma p (\vec{\nabla} \cdot \vec{v}_1) = 0, \quad (17)$$

$$\sigma \vec{B}_1 = \vec{e}_y V' B_{1z} + \vec{\nabla} \times (\vec{v}_1 \times \vec{B}) + \eta \Delta \vec{B}_1, \quad (18)$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0. \quad (19)$$

This set of equations determines the behavior of small perturbations.

### III. CRITERIA OF INSTABILITY

A general set of Eqs. (15)–(19) can be substantially simplified under our assumptions regarding the basic state. As it was mentioned, small departures from hydrostatic equilibrium in the basic state cannot influence the behavior of perturbations if inequality (14) is satisfied. Therefore the term proportional to  $\rho_1/\rho$  on the right-hand side (rhs) of Eq. (15) can be neglected since it is proportional to small departures from hydrostatic equilibrium. From Eq. (19), we have  $\partial B_{1z}/\partial z = 0$  and, hence,  $B_{1z} = 0$ . Then, the  $x$ ,  $y$ , and  $z$  components of the momentum equation are

$$\sigma v_{1x} = \frac{B_z}{4\pi\rho} \frac{\partial B_{1x}}{\partial z}, \quad (20)$$

$$\sigma v_{1y} + V' v_{1z} = \frac{B_z}{4\pi\rho} \frac{\partial B_{1y}}{\partial z}, \quad (21)$$

$$\sigma v_{1z} = \frac{c_s^2}{\sigma} \frac{\partial^2 v_{1z}}{\partial z^2} - \frac{1}{4\pi\rho} \frac{\partial}{\partial z} (B_y B_{1y}). \quad (22)$$

Equation (23) yields for the  $x$  and  $y$  components of the magnetic field

$$\left( \sigma - \eta \frac{\partial^2}{\partial z^2} \right) B_{1x} = B_z \frac{\partial v_{1x}}{\partial z}, \quad (23)$$

$$\left( \sigma - \eta \frac{\partial^2}{\partial z^2} \right) B_{1y} = B_z \frac{\partial v_{1y}}{\partial z} - \frac{\partial}{\partial z} (B_y v_{1z}). \quad (24)$$

Combining now Eqs. (21), (22), and (24) and taking into account that  $B_y \gg dB'_y$ , we can obtain the equation that contains only perturbation  $v_{1z}$ ,

$$\left\{ \left[ \sigma \left( \sigma - \eta \frac{\partial^2}{\partial z^2} \right) - c_{Az}^2 \frac{\partial^2}{\partial z^2} \right] \left( \sigma^2 - c_s^2 \frac{\partial^2}{\partial z^2} + \frac{B_y}{B_z} \sigma V' \right) - \frac{B_y}{B_z} \sigma^2 \left[ V' \left( \sigma - \eta \frac{\partial^2}{\partial z^2} \right) + \frac{B_y}{B_z} c_{Az}^2 \frac{\partial^2}{\partial z^2} \right] \right\} v_{1z} = 0, \quad (25)$$

where  $c_s^2 = \gamma p / \rho$ . This equation can be solved easily since all coefficients are approximately constant in our model. To solve Eq. (25), one needs the boundary conditions. Note that the eigenvalues are not very sensitive to the boundary conditions. Therefore we choose the simplest model conditions and assume that  $v_{1z} = 0$  at  $z = 0$  and  $z = d$ . Then,  $v_{1z} \propto \sin qz$  where  $q = \pi n / d$  and  $n$  is an integer. From Eq. (25), we have the following dispersion equation for the fundamental mode ( $n = 1$ ):

$$\sigma^4 + \sigma^3 \omega_\eta + \sigma^2 (\omega_s^2 + \omega_m^2) + \sigma (\omega_{BV}^3 + \omega_s^2 \omega_\eta) + \omega_{Az}^2 \omega_s^2 = 0, \quad (26)$$

where  $\omega_\eta = \eta q^2$ ,  $\omega_s = c_s q$ ,  $\omega_{Az} = c_{Az} q$ ,  $\omega_{BV}^3 = q^2 c_{Az} c_{Ay} V'$ , and  $\omega_m^2 = q^2 (c_{Ay}^2 + c_{Az}^2)$ ;  $c_{Ay}^2 = B_y^2 / 4\pi\rho$ . This equation describes fast and slow magnetosonic waves modified by shear.

The conditions under which Eq. (26) has unstable solutions can be obtained by making use of the Routh-Hurwitz theorem (see [18,19]). In the case of the dispersion equation of a fourth order, the Routh-Hurwitz criteria are written, for example, in [20]. According to these criteria, Eq. (26) has unstable solutions if one of the following inequalities is fulfilled:

$$\omega_\eta < 0, \quad \omega_{Az}^2 \omega_s^2 < 0, \quad (27)$$

$$\omega_{BV}^3 - \omega_\eta \omega_m^2 > 0, \quad (28)$$

$$\omega_{BV}^6 + \omega_{BV}^3 \omega_\eta (\omega_s^2 - \omega_m^2) - \omega_\eta \omega_s^2 \omega_{Ay}^2 > 0, \quad (29)$$

where  $\omega_{Ay}^2 = c_{Ay}^2 q^2$ . Two conditions (27) never apply because  $\omega_\eta$ ,  $\omega_{Az}^2$ , and  $\omega_s^2$  are positive. In the limit of small magnetic diffusivity, Eqs. (28) and (29) are equivalent to

$$\omega_{BV}^3 \neq 0, \quad (30)$$

that is the generalization of the condition derived by Bonanno and Urpin [17] for differentially rotating flows. Apart from shear, condition (30) requires nonvanishing  $y$  and  $z$  components of the magnetic field. The direction of  $B$  and the sign of  $V'$  are insignificant, and the instability may occur for both positive and negative  $V'$ . Note that the instability given by Eq. (30) can arise even in a very strong field.

Consider criteria (28) and (29) in the case when dissipation cannot be neglected. Condition (28) can be rewritten as

$$\frac{B_z}{B_y} > 2\pi^2 \left( \frac{2\eta}{d^2 V'} \right) \left( 1 + \frac{B_z^2}{B_y^2} \right). \quad (31)$$

As it was mentioned, our consideration is valid only if the condition  $B_y > V' B_z d^2 / 2\eta$  is satisfied [see Eq. (8)] that is equivalent to

$$\frac{B_z}{B_y} < \frac{2\eta}{d^2 V'}. \quad (32)$$

Since inequalities (31) and (32) are incompatible in the chosen longitudinal field, criterion (28) cannot be fulfilled and hence  $\omega_{BV}^3 - \omega_\eta \omega_m^2 < 0$ .

Since  $\omega_{BV}^3 - \omega_\eta \omega_m^2 < 0$  in the considered flow, we can transform criterion (29) into

$$\omega_{BV}^3 + \omega_\eta \omega_s^2 + \frac{\omega_\eta^2 \omega_s^2 \omega_{Az}^2}{\omega_{BV}^3 - \omega_\eta \omega_m^2} < 0. \quad (33)$$

Taking into account Eq. (32), we can estimate  $|\omega_{BV}^3 - \omega_\eta \omega_m^2| \sim \omega_\eta q^2 c_{Ay}^2$ . Then, the last term on the left-hand side (lhs) is of the order of  $\omega_\eta \omega_s^2 (B_z / B_y)^2$  and can be neglected compared to the second term. Hence criterion (33) is approximately equivalent to

$$\omega_{BV}^3 + \omega_\eta \omega_s^2 < 0. \quad (34)$$

This condition can be fulfilled only if

$$B_z B_y V' < 0, \quad (35)$$

that is the necessary condition of instability. In accordance with this condition, the imposed longitudinal field should have the same direction as the field stretched from  $B_z$ . If inequality (35) is satisfied, then the instability arises if  $|\omega_{BV}^3| > \omega_\eta \omega_s^2$ , or

$$\frac{c_{Ay}^2}{c_s^2} > 2\pi^2 \frac{B_y}{B_z} \frac{2\eta}{d^2 V'}. \quad (36)$$

This inequality can be fulfilled in a wide variety of strongly magnetized flows where the magnetic pressure is greater than the thermal pressure. Equations (35) and (36) determine the necessary and sufficient conditions of shear-driven instability in a Couette flow.

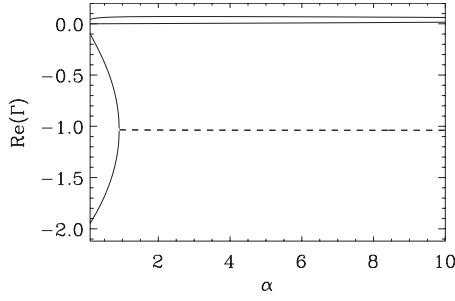


FIG. 1. The dependence of the real part of  $\Gamma$  on  $\alpha$  for  $\beta=0.01$ ,  $\epsilon=0.1$ , and  $\xi=2$ . Solid lines show the growth rate and frequency of the real roots, and the dashed line corresponds to the complex root.

#### IV. GROWTH RATE OF INSTABILITY

Since the necessary condition of instability is given by Eq. (35), we consider the roots of Eq. (26) only in the case of negative  $\omega_{BV}^3$  when  $\omega_{BV}^3 = -|\omega_{BV}^3|$ . To calculate the growth rate it is convenient to introduce dimensionless quantities,

$$\Gamma = \frac{\sigma}{|V'|}, \quad \xi = \frac{4\pi^2 \eta}{d^2 |V'|}, \quad \epsilon = \frac{B_z}{B_y},$$

$$\alpha = \frac{q^2 c_{Ay}^2}{|V'|^2}, \quad \beta = \frac{c_s^2}{c_{Ay}^2}.$$

Then, Eq. (26) becomes

$$\Gamma^4 + \Gamma^3 \xi + \Gamma^2 \alpha (1 + \epsilon^2 + \beta) + \Gamma \alpha (\beta \xi - \epsilon) + \epsilon^2 \beta \alpha^2 = 0. \quad (37)$$

This equation was solved numerically for different values of the parameters by computing the eigenvalues of the matrix whose characteristic polynomial is given by Eq. (26) (see [21] for details). Moreover it is not difficult to see that in order to satisfy the constraint [Eq. (8)] we must choose  $\epsilon \ll 1$ .

In Fig. 1, we plot the dependence of real roots and the real part of complex roots on  $\alpha$  for  $\epsilon=0.1$ ,  $\xi=2$ , and  $\beta=0.01$ . The solid lines show roots when they are real, and the dashed line shows the real part of complex roots. Our calculations clearly indicate that two real roots are positive for the considered parameters and hence there should exist a shear-driven instability. The pair of complex roots split into a pair of real ones at  $\alpha \approx 0.9$ , but these roots always correspond to stable modes. One unstable root is rather large with the growth rate  $\sim 0.05-0.08V'$ , and another one is typically about ten times smaller. For these roots, the growth rate varies very slowly with the parameter  $\alpha$ . Only if the Alfvén frequency is smaller than the characteristic shear frequency and  $\alpha < 1$ , the growth rate of the most unstable mode decreases. Note that the considered instability occurs at a very large magnetic pressure that exceeds the gas pressure by two orders of magnitude.

In Fig. 2, we plot the same dependence as in Fig. 1 but for  $\xi=10$ . The higher value of  $\xi$  corresponds to a larger magnetic viscosity and hence to a stronger dissipation of perturbations. Due to this, the instability turns out to be suppressed. Indeed, all roots are either negative or have a negative real part. In this case, complex conjugate roots have a very small nega-

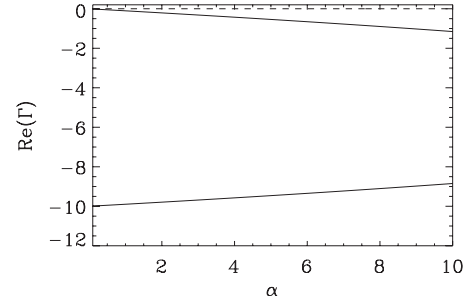


FIG. 2. The same as in Fig. 1 but for  $\xi=10$ .

tive part, but real roots dissipate much more rapidly. Note that the critical value  $\xi$  that discriminates between stable and unstable flows is  $\sim 10$ , and the instability occurs if  $\xi < 10$ . For example, the growth rate can reach  $\sim 0.04V'$  in a flow with  $\xi=5$ .

Figure 3 shows the dependence of a real part of  $\Gamma$  on  $\alpha$  for  $\beta=0.01$ ,  $\epsilon=0.03$ , and  $\xi=2$ . The right panel shows the behavior of roots at the top left region of the left panel where roots are small. Comparing with Fig. 1, it is seen that a decrease of the ratio  $\epsilon=B_z/B_y$  results naturally in a smaller growth rate. This dependence is qualitatively clear since the considered instability is due to the presence of a transverse field component in a flow and, therefore, a decrease of this component leads to a weaker instability. In the considered range of  $\alpha$ , both oscillatory and nonoscillatory modes can arise. Two nonoscillatory modes are unstable if  $\alpha < 2.8$ . After merging, this couple forms a pair of complex conjugate modes that are unstable if  $\alpha > 2.8$ . The growth rate of one nonoscillatory is larger than that of oscillatory modes and can reach  $\sim 0.01V'$ . The growth rate of oscillatory modes is a factor of  $\sim 2$  smaller. Note that another couple of modes is always stable.

In Fig. 4, we plot the dependence of a real part of  $\Gamma$  on  $\alpha$  for  $\epsilon=0.5$ ,  $\xi=30$ , and  $\beta=0.01$ . We show only three roots in this figure since the fourth root has a large negative value,  $\Gamma \sim \xi \sim -30$ . The increase of  $\epsilon$  leads to a corresponding increase in the growth rate. Like the previous case, the instability can arise either in oscillatory or nonoscillatory regimes. Two nonoscillatory modes are unstable if  $\alpha < 2$ . After merging at  $\alpha \approx 2$ , these real roots form a couple of complex conjugate roots that are unstable at  $\alpha > 2$ . Another pair of

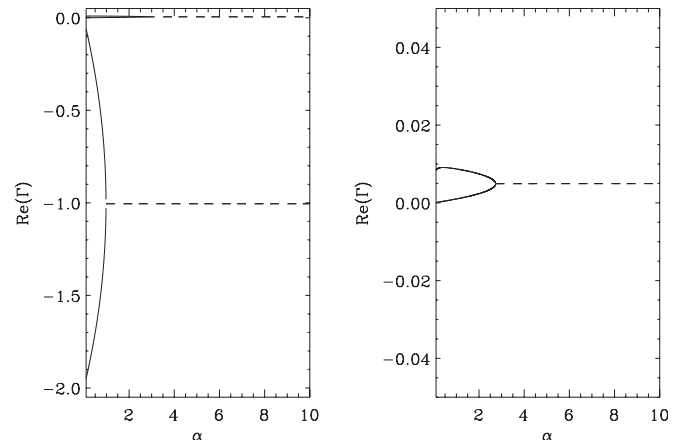


FIG. 3. The same as in Fig. 1 but for  $\epsilon=0.03$ .



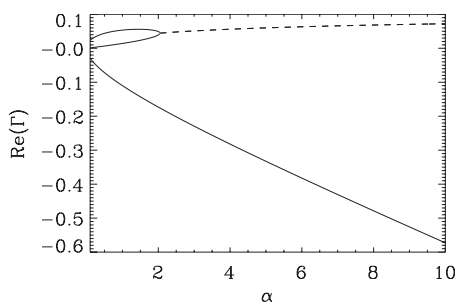


FIG. 4. The dependence of a real part of  $\Gamma$  on  $\alpha$  for  $\epsilon=0.5$ ,  $\xi=30$ , and  $\beta=0.01$ .

modes is always stable. The growth rate of unstable modes is  $\sim 0.07-0.1V'$  and increases slightly with  $\alpha$ . Note that oscillatory modes can grow faster than nonoscillatory ones in this case.

Figure 5 shows the growth rate of instability for a very small value of  $\beta=0.001$ . This  $\beta$  corresponds to the magnetic pressure approximately three orders of magnitude greater than the gas pressure. Despite a very high magnetic pressure, the instability can still occur. This is in agreement with our analytic result that the instability should not be suppressed by a strong magnetic field. The dependences in Fig. 5 are qualitatively very similar to those shown in Fig. 4. Two nonoscillatory modes are unstable in this case as well. One unstable mode has a very small growth rate  $\sim 0.01V'$ , but another one grows much faster,  $\sigma \sim 0.1V'$ . The growth rate of the fastest growing mode is even higher than in the case  $\beta=0.01$  despite a strong magnetic field.

## V. DISCUSSION

To summarize then, we have considered the instability caused by shear in a compressible magnetized gas. To illustrate the main qualitative features of the instability associated to compressibility and shear, we analyzed a particular case of perturbations that depend on the vertical coordinate alone. The plane Couette flow with a nonvanishing transverse magnetic field turns out to be unstable even in this simplest case. The necessary condition of instability is  $B_z B_y V' < 0$ , and it can be easily satisfied in laboratory flows. Since the shear flow in the presence of a transverse magnetic field always stretches the longitudinal field satisfying the necessary condition (35), one can expect that the instability likely operates if  $B_y$  is entirely generated by shear and  $C_1=C_2=0$  in Eq. (8). We consider this case elsewhere.

The newly found instability is relatively slow: its growth rate reaches  $\sim 0.1V'$  and is small compared to the shear time scale,  $1/V'$ . However, even this growth rate can be sufficient to generate hydrodynamic motions in many real flows, for

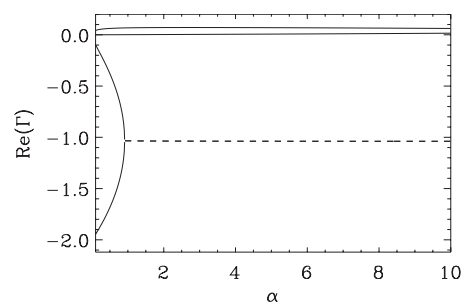


FIG. 5. The dependence of a real part of  $\Gamma$  on  $\alpha$  for  $\epsilon=0.1$ ,  $\xi=2$ , and  $\beta=0.001$ .

example, in astrophysics. Basically, the growth rate is larger for nonoscillatory modes which are unstable at relatively not very large  $\alpha$ . The growth rate depends on the ratio of the magnetic and gas pressure, being smaller for a low ratio.

The considered instability is related basically to shear and compressible properties of a magnetized gas. In the incompressible limit that corresponds to  $c_s \rightarrow \infty$ , we have from Eq. (26)

$$\sigma^2 + \sigma\omega_\eta + \omega_{Az}^2 = 0, \quad (38)$$

and the instability does not occur for chosen perturbations. It can be not the case, however, for perturbations of a more general form which depend also on the  $x$  or  $y$  coordinates.

This instability can be either oscillatory or nonoscillatory, depending on the value of the ratio  $2\pi c_{Ay}/dV'$ . Typically, the considered instability is nonoscillatory if  $\alpha$  is not large and oscillatory in the opposite case. The critical  $\alpha$  that determines the transition between oscillatory and nonoscillatory regimes depends strongly on the parameters  $\epsilon$ ,  $\xi$ , and  $\beta$  and can vary within a wide range.

One more important feature of the instability is associated with the dependence on the magnetic field strength. Generally, a sufficiently strong magnetic field can suppress instabilities of a shear flow. On the contrary, the instability in our study cannot be suppressed even in very strong magnetic fields as it is seen from the criterion (40). All this comparison allows us to claim that our analysis demonstrates the presence of an instability in compressible shear flows.

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